Forward Problems and their Inverse Solutions

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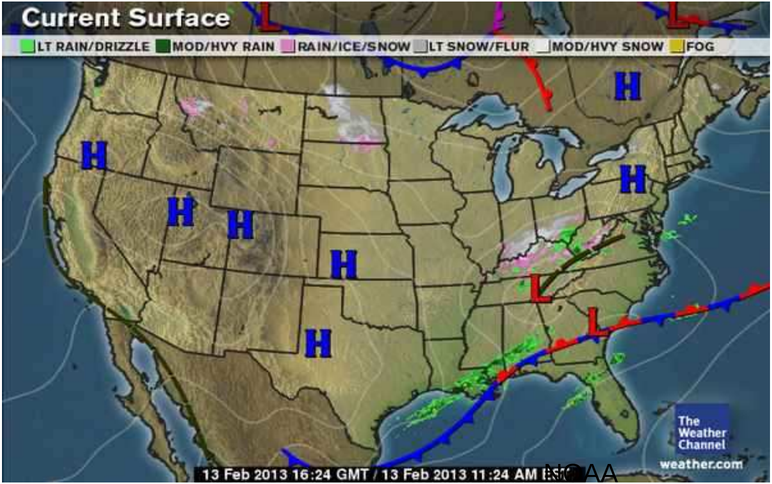
Outline

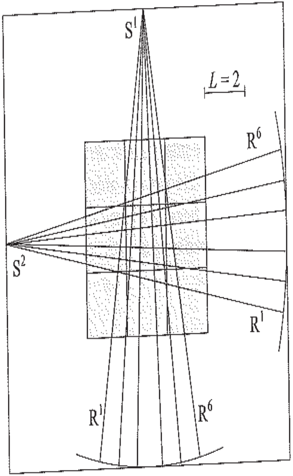
Weather Forecasting



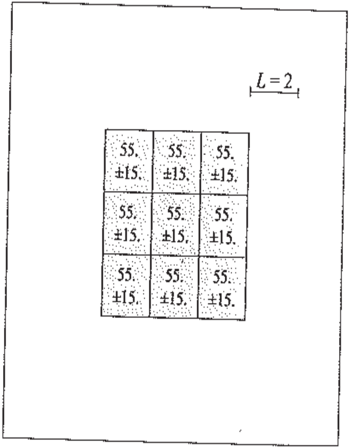
NOAA

CT SCAN (1/4)

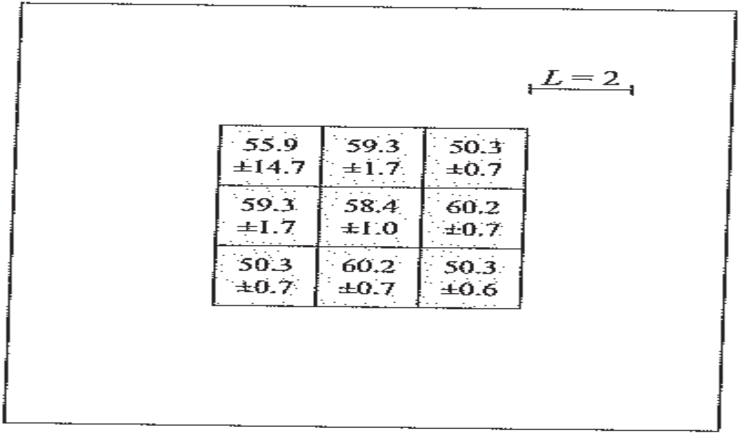
Setup:



InitialGuess:

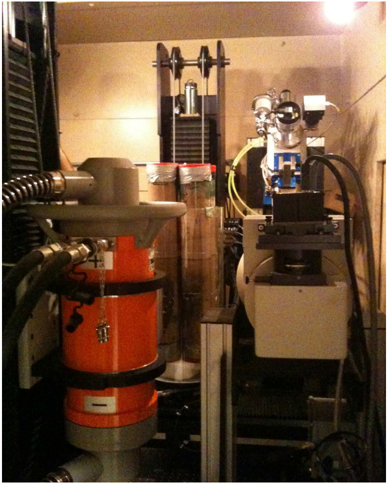


CT SCAN (2/4)

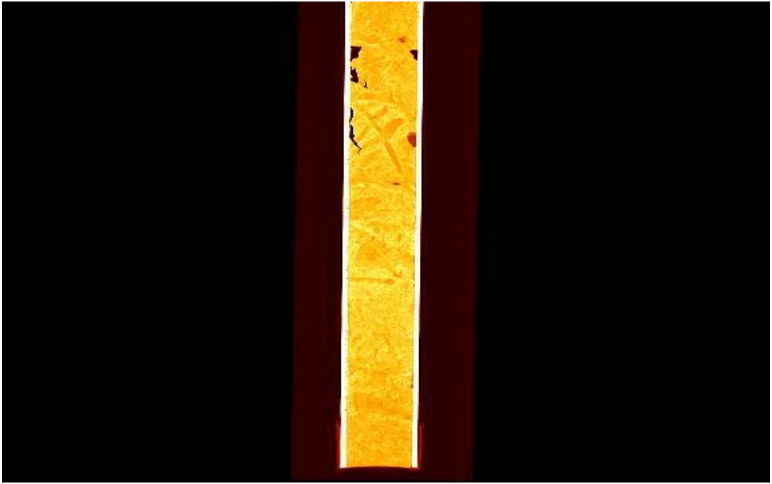


Source: Tarantola, 2005

CT SCAN (3/4)



CT SCAN (4/4)

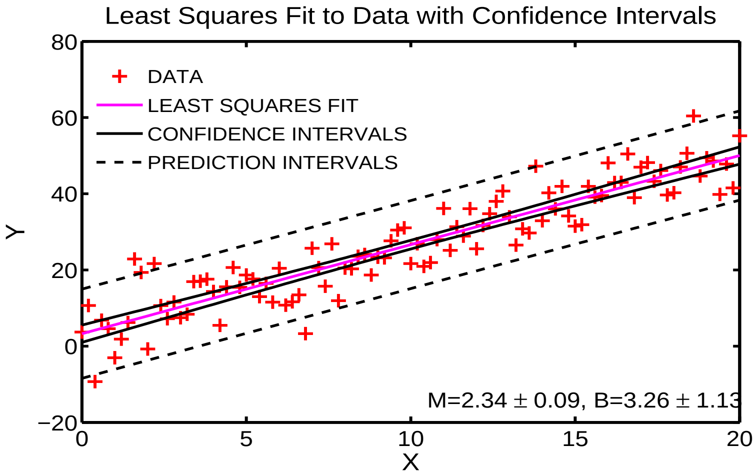


L.Stott

"Simple" Case of Line Fit

- A simple inverse problem is "y=mx". Instead of knowing m and x to solve for y, you solve for either x or m given knowledge of y and m or y and x.
- What we can easily measure is often not what we want to know.
- In an inverse problem, we use observations to infer what we want to know.

Least Squares



Bayesian Methods

- Part 1: Direct Method
- Part 2: Sampling Method (MCMC)

Bayesian vs. Least Squares

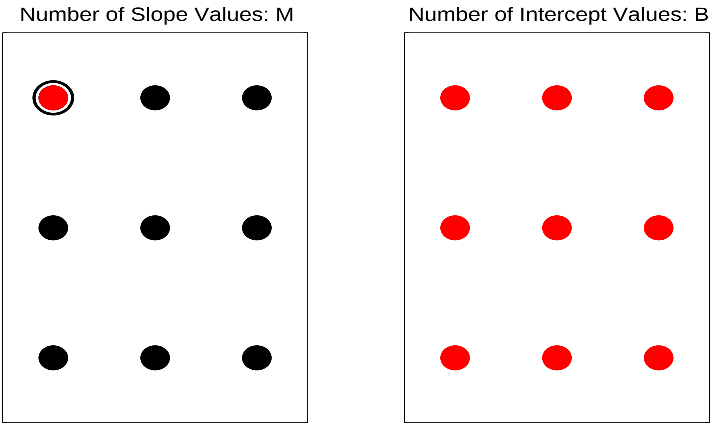
- start with initial guess
- solve for a distribution on the parameters (many solutions)

Bayesian

- initial guess of parameters is called the prior distribution (shapes solution)
- "I know almost nothing about the parameters" –> assume uniformly distributed

Part 1: DIRECT METHOD

Direct Method



Number of Calculations: MxB=81

Direct Method

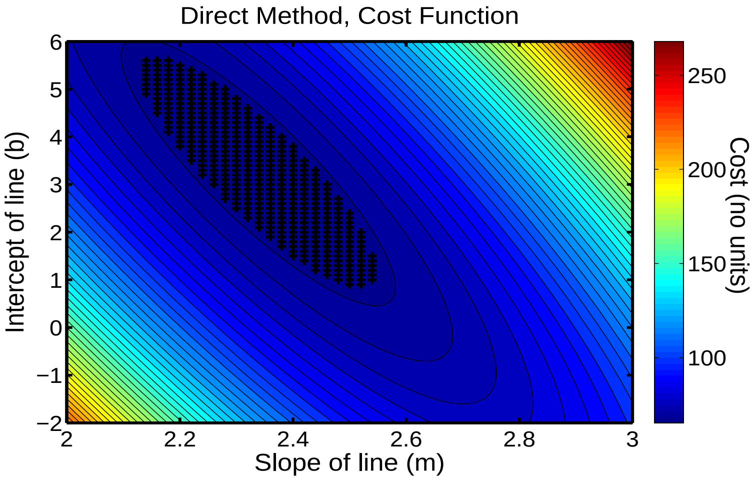
Finding the best solutions:

- Define a cost function:

$$2J(m,b) = (g(m) - d)^T C^{-1} (g(m) - d) = \sum_{1}^{N} \frac{((mx + b) - y)^2}{\sigma^2} \quad (1)$$

- best solutions are where cost is within 2σ of the mean (95% confidence interval)

Direct Method



Direct Method

Finding the best solutions:

- Define a cost function:

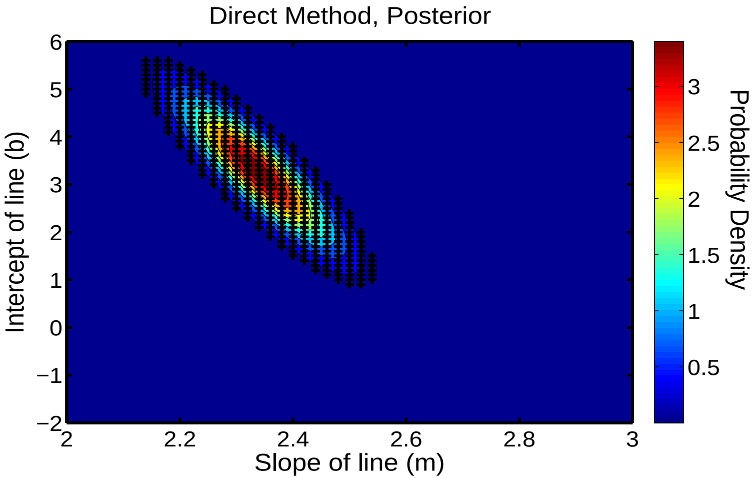
$$2J(m,b) = (g(m)-d)^T C^{-1}(g(m)-d) = \sum_1^N \frac{((mx+b)-y)^2}{\sigma^2} \tag{2}$$

- best solutions are where cost is within 2σ of the mean (95% confidence interval)
-

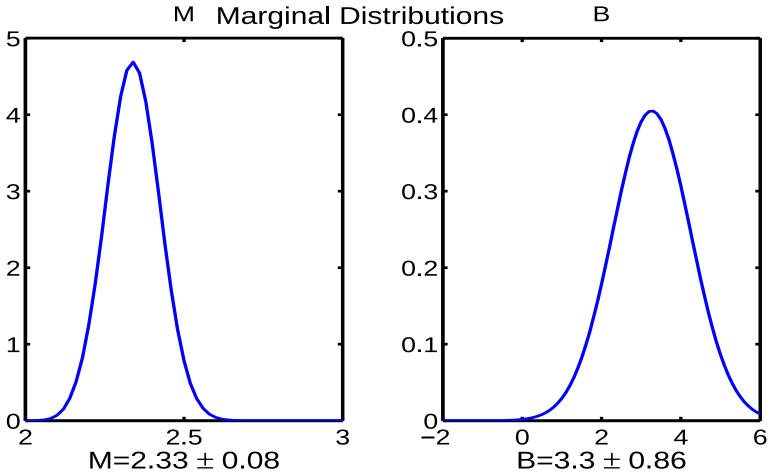
$$P(m,b) = e^{-J(m,b)} \tag{3}$$

is the posterior distribution on the parameters (central limit theorem)

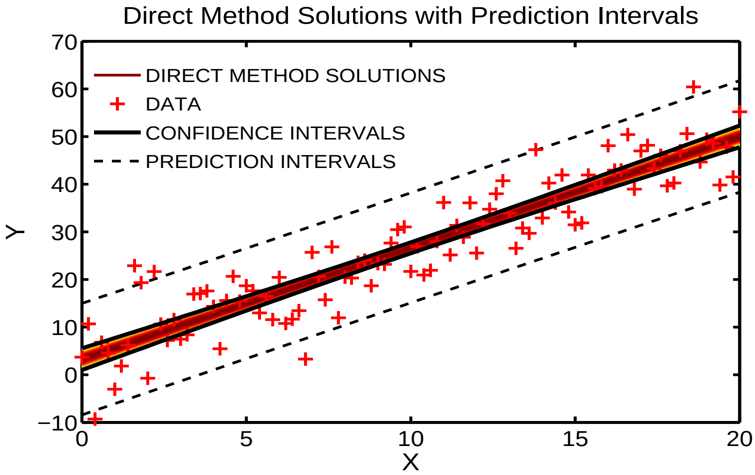
Direct Method



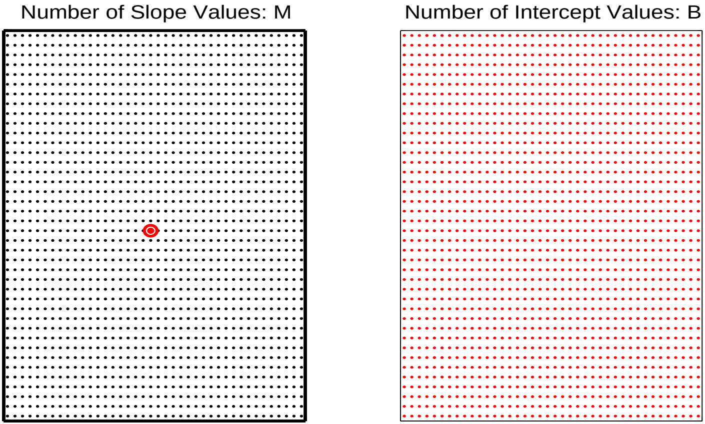
Direct Method



Direct Method



Direct Method



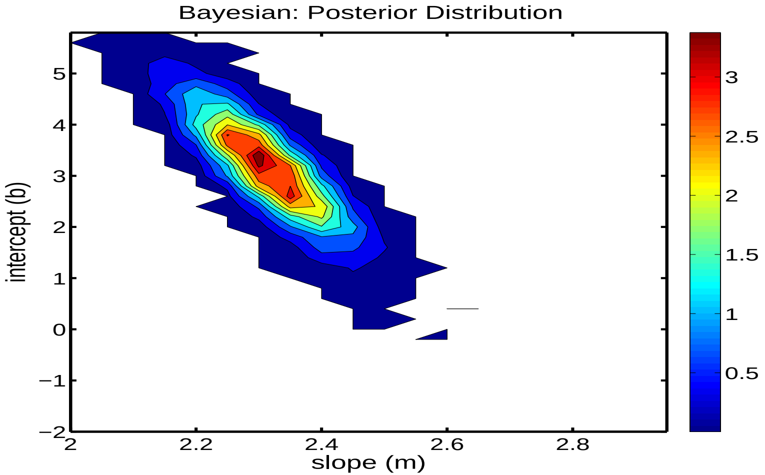
Number of Calculations: MxB=$2.56*10^6$

Part 2: Sampling Method (MCMC)

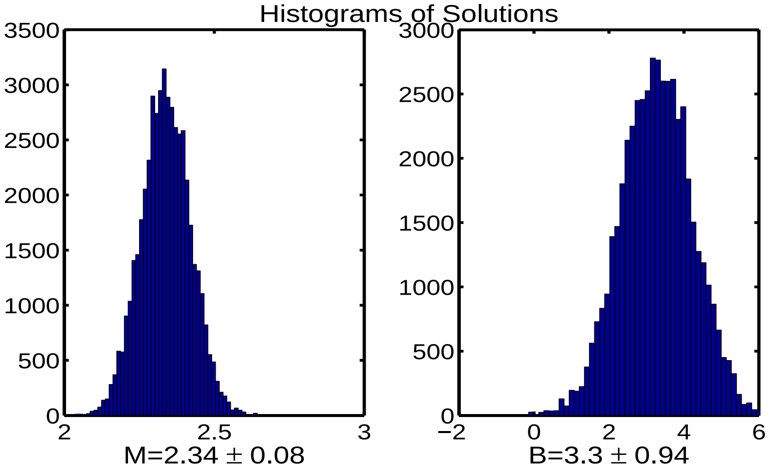
Metropolis-Hastings: Monte Carlo Markov Chain Algorithm

- **Step 1: Start with** (m_0, b_0)
- **Step 2: Step a random distance to** (m_1, b_1)
- **Step 3: If** $P(m_1, b_1) < P(m_0, b_0)$ **accept** (m_1, b_1)**.**
- **Step 4: If not, calculate a transition probability** $P(m_1, b_1)/P(m_0, b_0)$**. Calculate a random number** ϵ **between 0 and 1. If** $P(m_1, b_1)/P(m_0, b_0) >= \epsilon$**, accept** (m_1, b_1)**. Otherwise, reject.**
- **Step 5: If** (m_1, b_1) **is accepted, start Step 2 at** (m_1, b_1)**. Otherwise, start Step 2 at** (m_0, b_0)

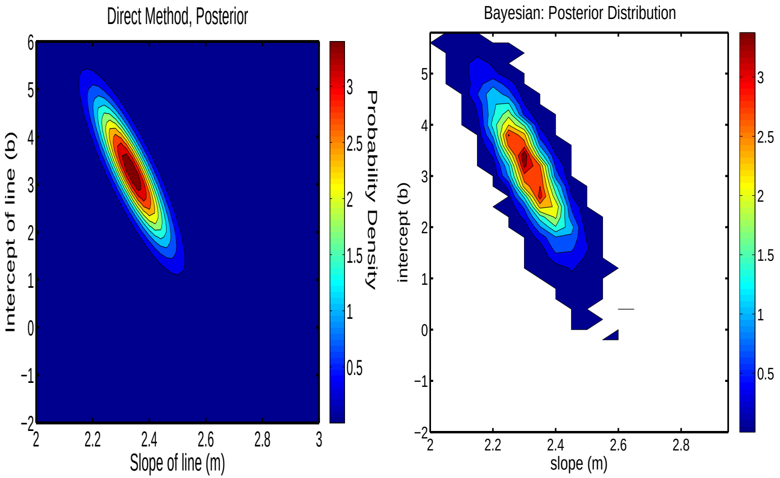
Metropolis-Hastings



Metropolis-Hastings



Metropolis-Hastings



Summary

- Least squares: maximum likelihood solution with an error bar.
- Direct methods: could miss mass in posterior distributions, computationally expensive
- Sampler methods: don't cover all parameter space (can miss multiple peaks in cost function), easier on computer

Research Applications

Current work:

- estimation of wind stress under hurricane forcing conditions from consideration of observed ocean response
- constraining constant parameters in turbulent mixing models to observations of ocean state in tropical Pacific

Future research:

- sampler used (Jackson et al., 2004)
- definition of the cost function (Mu et al., 2004)
- development of solution smoothers

Acknowledgements

Thanks to Deborah Khider and Charles Jackson for help with all aspects of putting this talk together.